

**SPECIFIC ASPECTS OF TEACHING MATHEMATICS TO PERSONS
PROFESSIONALLY STUDYING PHYSICS**

**ASPECTE SPECIFICE ALE PREDĂRII MATEMATICII PERSOANELOR
CARE STUDIAZĂ FIZICA LA NIVEL PROFESIONAL**

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Rezumat. Scopul lucrării este de a descrie diferențele specifice, caracteristicile, tehnicile metodologice și abordările care ar trebui folosite atunci când se predă matematica persoanelor care sunt interesate de fizică și au posibilitatea de a o studia profesional. Vorbind despre abilități, ne bazăm pe o abordare bazată pe activitate, a cărei esență este că prezența (sau absența) anumitor abilități este indicată, în primul rând, de procesul și rezultatul activității corespunzătoare. La rezolvarea unei probleme din punct de vedere fizic, este necesar, în primul rând, să se determine, să se aleagă sau să se construiască independent un model fizic al procesului, fenomenului real luat în considerare în problemă. În continuare, trebuie să se aleagă sau să se construiască în mod independent un model matematic care descrie modelul fizic corespunzător, adică să se determine aparatul matematic folosit pentru a rezolva problema și a lega modelele între ele. Persoanele care studiază fizica în mod intenționat prețuiesc cunoștințele matematice, respectă matematica ca știință, dar percep aceste cunoștințe foarte utile, ca un mijloc de înțelegere a științei fizice. Prin urmare, atunci când studiem materialul de matematică, este recomandabil să se demonstreze posibilitatea aplicării sale practice.

Cuvinte-cheie: învățare, fizică, matematică, abilități, activitate, dezvoltare, sarcină.

Statement and justification of the relevance of the problem. Mastery of physical science is impossible without mastering thorough mathematical knowledge and corresponding skills. At the same time, the specificity of such knowledge and skills lies in their predominantly applied nature. That is, the attitude towards the study of mathematics of persons studying physics, as an important component of future professional activity, differs, in our opinion, firstly, in that, for pragmatic reasons, they master mathematics as a tool, a means of mastering physical science, and secondly, by the fact that the styles of thinking, and consequently the

processes of perception, transformation of information and approaches to setting and solving actual problems among “mathematicians” and “physicists,” although they have much in common, are at the same time significantly different.

At the stage of obtaining and perceiving primary information, these differences lie in the main approaches to the primary analysis of scientific information and determining its most important and priority component from a physical point of view. In other words, the same scientific or technical information is perceived differently by a “mathematician” and a “physicist”, because in fact it has a slightly different meaning for them. At the stage of goal-setting and searching for ways to solve applied problems or problems, differences in approaches also, as a rule, appear due to existing differences in the way different people comprehend information, and, consequently, between people’s mathematical abilities and abilities to study physics.

Analysis of the latest research and publications. The content of mathematics abilities, as well as their structure, at one time was described in detail in psychological and pedagogical literature [1]. At the same time, we note that the classical consideration of mathematical abilities was carried out mainly from a psychological point of view, and we are primarily interested in the methodological aspects, differences and features of teaching mathematics and physics, in particular, teaching mathematics to those who are professionally interested in physics. The theoretical aspects of the psychological and pedagogical problem of having the ability to study physics, as well as their differences from mathematical abilities, have been discussed in detail by us in a number of publications, in particular in [2].

Purpose of the article. The purpose of the work is to identify and describe specific differences, features, methodological techniques and approaches that it is advisable to have in the pedagogical arsenal and use when teaching mathematics to people interested in physics and having the ability to study it.

Presentation of the main research material. Based on the theoretical foundations for considering and studying the concept of abilities in general, we highlight two fundamental points that, in our opinion, allow us to determine and describe the characteristics of teaching mathematics to people who are interested and have the ability to study physics.

Firstly, as the experience of our scientific and pedagogical work allows us to assert, there is no point in dividing, that is, considering separately the abilities to study physics among high school students (students), students and young professionals (bachelors, masters, graduate students). We can talk about significant differences regarding the level and depth of their knowledge, skills and experience in conducting scientific research, but the style of thinking and ways of perceiving, assimilating and transforming scientific information in their minds are, in principle, very similar.

Secondly, one way or another, all existing differences between the abilities to study physics and the abilities to study mathematics are predetermined, explained and further determined by the fact that physics is inherently an experimental science that uses mathematics and its capabilities as an idealized tool (means) research. At the same time, the fact remains indisputable that abilities in both sciences have a lot in common.

Research methods. Speaking about abilities, we are based on the “activity” principle or approach, the essence of which is that the presence (or absence) of certain abilities is evidenced, first of all, by the process and result of the individual’s corresponding activity. For example, it

is obvious that with regard to musical abilities, the conclusion about their presence is made only on the basis of certain achievements of the individual in the musical field, that is, playing musical instruments, reproducing melodies from memory, composition, etc.

Likewise, due to the effectiveness of the relevant activity, the process of manifestation of artistic, literary, and other abilities in the field of art or intellectual activity occurs. In this sense, the identification and development of intellectual and artistic abilities are very similar in form and the main difference between them is determined, as a rule, by the content of the activity. At the same time, the very approach to setting and searching for ways to solve a specific problem may indicate the presence of certain abilities, inclinations and the formation of a certain style of thinking.

Since, as already noted, abilities in physics and mathematics have much in common, the question arises of how to recognize which abilities prevail, under what conditions and when, and what exactly such abilities should be considered.

As a result of a long analysis, comprehension and generalization of the accumulated experience in organizing and conducting intellectual competitions in physics, we propose an approach that allows us to clearly distinguish individuals who have predominantly mathematical abilities from those who have an ability in physics. It is interesting and useful that the proposed criterion works already at the stage of performing theoretical tasks, even without performing experimental physical studies, laboratory work, etc. Note that abilities in physics and mathematical abilities are in no way antagonistic, but we are talking about the dominant, predominant direction of an individual's intellectual activity.

We have repeatedly had to examine situations that arise, in particular, when participants in physics olympiads at different levels perform tasks: from regional to international. At each of the corresponding stages of the competition, we observed a similar situation. Participants in the competition, who had completed approximately the same course of mathematical training, solving problems in physics, en masse and almost synchronously completed tasks to a certain and predictable level, after which they stopped. Appropriate progress in solving a problem in physics was possible through a fairly formal application of mathematical knowledge and computing apparatus (sometimes even of a very high level) and a fairly formal application of basic knowledge in physics. At the same time, difficulties arose in solving the problem, as soon as the participants in physics competitions who were solving the problem had the need to apply physics knowledge informally. In other words, there was a need to select and apply a certain original, non-standard (that is, directly and formally not clearly derived from anywhere) physical technique, hypothesis, assumption, approximation, neglect of some parameters, characteristics or quantities, etc. In such cases, there was often a fairly sharp distribution of all participants in the competition into two, usually unequal, parts.

Most of the participants actually stopped solving having received a certain intermediate result, achieved by using the data of the problem conditions and the formal application of the mathematical apparatus, and a minority of the participants, having made the necessary non-standard move from a physical point of view, successfully moved on to the next similar "trap", where in principle, the process of distributing rivals was repeated in approximately the same proportions.

In our opinion, it is also interesting that if we analyze the degree of individual progress of the competition participants in solving various problems (and at physics Olympiads there are

usually from three to five), then we can notice a high correlation of results (percentages of completing tasks) in completing various tasks by the same competitor. In other words, if a particular individual, when solving a problem, acts based primarily on formal procedures and uses the capabilities of the mathematical apparatus involved, then this gives him approximately the same result for solving different problems of different topics and directions.

In this case, we are talking about the presence of, at a minimum, mathematical abilities, but to state the presence of abilities in physics, this fact, from our point of view, is not enough. To do this, it is necessary to obtain convincing results achieved by using original methods and solutions, and then, based on the principle of an “activity” approach to determining talent, analyzing the significance of the results achieved, you can try to qualitatively assess (compare) the level of abilities in physics of different young people.

Mathematics is based on the use of abstract concepts, the construction of a priori ideal models and the search for the shortest (simplest) solutions. This, in particular, is the essence and value of mathematical science, but the paradox of using mathematical knowledge lies in the fact that in order to solve problems in physics, the mathematical apparatus must not only be used, but also be able to optimize it, and sometimes consciously limit this use.

We will deliberately not focus on such specific subjects as “solving differential equations” or “methods of mathematical physics”. These courses are a continuation of the theory of solving differential equations, which occupy a kind of “intermediate level” in the structure of physical and mathematical knowledge, and in terms of approaches and methods of study, they are quite close to the study of mathematical disciplines, since they significantly relate to mathematical approaches and methods for solving the already described constructions (from a mathematical point of view) equations, that is, idealized models of physical processes. In the general case, the mathematical approach to the solution in a certain sense is “uncompromising”, strictly formalized, and a completely satisfactory result of solving a mathematical problem from a physical point of view often turns out to be meaningless, abstract, “ideal” and therefore not always connected with reality and be so, which can be considered a comprehensive solution to the physical problem.

For example, from the point of view of mathematics, there is nothing surprising in obtaining a negative value of time, negative absolute temperature, or theoretical consideration of any n -dimensional space, etc. That is, the obtained mathematical solution to a physical problem, as a rule, requires refinement or interpretation, taking into account the physical content of the solution, and only after that the solution obtained mathematically will become the final solution to the physical problem.

This approach is quite normal and justified from a scientific and practical point of view, but certain difficulties can sometimes arise at the stage when certain simplifications, substitutions or assumptions need to be made directly at the stage of performing a mathematical solution. In such cases, because the mathematical solution to the problem in a more general form is either too complex or unnecessary at such a too high level of generalization, the need for a specific solution may disappear altogether.

Sometimes methods that, with a very strict approach, may seem somewhat “vulgar” from the point of view of “pure” mathematics, in practice turn out to be quite effective and efficient, since they reflect the physical content of the problem and at the same time simplify the solution process.

For example, to construct and describe mathematically a physical model of rain in general, if possible, then this model will turn out to be extremely complex and, as a result, cannot be used effectively enough. At the same time, if we specify the problem regarding exactly what processes and their consequences associated with rainfall are of interest to researchers, then the problem can be simplified and its solution can be made (paradoxically) more complete and comprehensive. If the problem of rainfall in a certain region is being studied in order to build storm drains and avoid flooding of territories or aquaplaning of cars on the roads, this is one task. If the problem arises of ensuring the construction of house roofs that effectively protect against water penetration, in particular, for example, in the case of strong side winds and strong lateral (inclined) water flows, etc., then this is a completely different problem about rain. If the problem arises of studying rain for its occurrence, namely the formation of drops, which, when falling from a great height, can damage plantings, etc. then this is a different task. If we study the problem regarding how to effectively “disperse” unwanted rain clouds, then this is already the third problem about rain and there can be many such problems from a physical point of view. Mathematical approaches and rain models, obviously, should be quite different in each case.

The physical approach to solving any problem consists of understanding its physical content, building a model that embodies this physical content, and using a formalized mathematical apparatus to find a solution to a problem that has an acceptable physical content. In other words, the solution is an explanation and practical useful application, built on the basis and from the point of view of accepted fundamental physical theories.

In the modern, diverse, multifaceted, rapidly changing world, which has many distracting temptations, the motivation of certain activities is becoming increasingly important. Accordingly, teaching mathematics, as well as any other training aimed at achieving maximum efficiency of the process, must be modern. And this means taking into account, in particular, the specifics of the student population, their cognitive needs, interests, abilities, inclinations, opportunities to learn and at the same time target, attract and encourage their cognitive activity, that is, motivate.

Young people who have an aptitude for physics and an interest in studying it are, as a rule, not interested in studying overly idealized or abstract issues. The tendency to study physics, in particular, lies in the desire to deal with issues that have physical content and practical meaning. That is, the study of physics (even theoretical) necessarily turns out to be connected with practical activity, since physical knowledge under any conditions differs from other knowledge in that it has physical content and is thus associated with the description of states, processes and natural phenomena. and the surrounding world in general.

Physics, as a science of nature, inherently strives, as accurately as possible, to describe natural (real) states, processes and phenomena using idealized models and approximations. When solving a problem from a physical point of view, it is necessary, first of all, to determine, select or independently construct a physical model of the real process, phenomenon, etc. being considered in the problem. Next, you need to choose or independently construct a mathematical model that describes the corresponding physical model, that is, decide on the mathematical apparatus used to solve the problem and link the models to each other.

At the same time, at the stage of selecting or creating physical and mathematical models and linking them, one should keep in mind at least two fundamental, from the point of view of physics, points:

- firstly, there is no point in simplifying the physical model as much as possible and striving for the most simple, from a mathematical point of view, solution to the problem, because “you can throw out the baby with the bathwater.” The physical model must reflect fundamentally important provisions, that is, the physical content of the problem;
- secondly, the selected mathematical model and the mathematical apparatus used must provide the possibility of solving the problem (obtaining the final solution) and be understandable and feasible for those who carry out this solution.

Thus, the process of solving a problem from a physical point of view is almost always a search for a reasonable (acceptable) compromise (balance) between “incomprehensible reality” and “non-existent ideality”, i.e. in essence, this is the problem of posing a problem to a real (existing in nature) problem.

Conclusions of the study and prospects for further developments. Based on a comparison of these two rather different approaches, we highlight those features that should be taken into account and those methodological techniques that are advisable to use when teaching mathematics to people inclined to study physics and having the appropriate thinking style, namely:

- Persons who purposefully study physics value mathematical knowledge, treat mathematics with respect as a science, but perceive this knowledge in a very utilitarian way, as a means of comprehending physical science. Therefore, when studying mathematics material with them, it is advisable to show the possibility of its practical implementation.
- Students with an aptitude for physics are not very keen, for example, on describing abstract and “unreal” n -dimensional spaces, but they perceive and assimilate much better material filled with physical content, supported by real examples of connections with reality.
- Physical examples that “revive” the mathematical theory should be given in a timely manner “on fresh tracks”, in one block with the study of the corresponding theory, explaining and supporting it.
- To develop the skills and abilities of physical research, it is useful to pose and solve problems, including in mathematics, that not only have several different ways or methods of solving them, but also allow the possibility of choosing, for example, boundary conditions, etc. fills them with physical content.

BIBLIOGRAPHY

1. Kreminskyi, B. G. *Abilities for physics: structure, content, development* / B. G. Kreminskyi // Our school. – 2009. – No. 6. – P. 7–13.
2. Kreminskyi, B. G. *Theory and practice of working with intellectually gifted pupils and students in physics*: Monograph / B. G. Kreminskyi. – K.: Nat. ped. University named after M. P. Dragomanova, 2011. – 421 p.