NEW APPROACH TO STUDYING MATHEMATICS IN THE 7TH GRADE WITHIN THE NEW UKRAINIAN SCHOOL PROJECT

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Rezumat. În articol este prezentată o nouă abordare privind studiul matematicii în clasa a VII-a în cadrul proiectului reformei educaționale „Noua școală din Ucraina". În special, sunt prezentate aspectele metodologice ale manualului de matematică pentru clasa a VII-a, elaborat de echipa de autori: *Oleksandr Shkolnyi, Yevhen Nelin, Andrii Mylianyk, Yulia Prostakova. Se pune accent pe evidențierea diferențelor fundamentale dintre manualul elaborat, axat pe modelul de program educațional propus de această echipă de autori, și manualele tradiționale de matematică, editate în Ucraina pentru clasa a VII-a.*

Cuvinte cheie: *Noua Școală Ucraineană, noua abordare a studiului matematicii, model de program educațional, manual de matematică pentru clasa a VII-a, caracteristici metodologice.*

Resume. *In the article we consider a new approach in studying math in 7th grade within the educational reform project "New Ukrainian School". Particularly, we observe the methodological features of the mathematics textbook for the 7th grade of the author's team consisting of Oleksandr Shkolnyi, Yevhen Nelin, Andrii Mylianyk, Yulia Prostakova. Emphasis is placed on the fundamental differences of this textbook, based on the model educational program of the same authors, from traditional Ukrainian mathematics textbooks for the 7th grade of secondary school.*

Keywords: *New Ukrainian School; new approach to studying math; model educational program; textbook on mathematics for the 7th grade; methodological features.*

Introduction. Within the framework of the „New Ukrainian School" educational reform project, fundamental approaches to the study of most school subjects, in particular, to the study of mathematics, are changing. The State Standard of Basic Secondary Education [1] determines the requirements for the competences of students in grades 5-9, model educational program determine the content of each of the school's educational disciplines. Textbooks are traditionally created according to model program. Finally, working program developed by teachers on the basis of model program are approved by the pedagogical councils of schools. The teacher, within the limits of his academic freedom and the realization of the right to professional self-realization in the work program, can make changes to the model program: change topics, expand and deepen information, etc. However, mathematics textbooks are mostly

created based on model programs. Therefore, teachers should take this into account when drawing up work programs and in their further work.

For example, let's consider the peculiarities of studying mathematics in the 7th grade according to the textbook of the author's team consisting of: Oleksandr Shkolnyi, Yevhen Nelin, Andrii Mylianyk, Yulia Prostakova. According to the authors' idea, this textbook on mathematics for the 7th grade should contribute to ensuring the proper quality of education of seventh-graders and the implementation of the provisions of the State Standard of Basic Secondary Education, as well as the model program in mathematics [2] within the framework of the New Ukrainian School reform. Let's briefly dwell on some of the principles reflected in the construction of a model program and the writing a textbook.

Main results. *First*, in the 7th grade, as in grades 1 to 6, we continue to study the mathematics course without dividing it into algebra and geometry. In this way, we follow the European and world tradition, according to which mathematics is mostly studied in this way. In addition, in this way we avoid certain terminological and logical inaccuracies caused by the division of mathematics into algebra and geometry, as well as certain inconsistencies between these courses that sometimes arise.

For example, the information about functions that are currently studied in the course of algebra obviously does not apply to algebra as a science, and the information about the equation of a straight line and circle, which is currently present in the course of geometry, is difficult to classify as purely geometric. It is also difficult to say which is more important (algebra or geometry) in the following problem: "Point C belongs to segment AB, and the length of segment AC is 3 cm greater than the length of segment BC. Find the lengths of segments AC and BC if $AB = 10$ cm." Obviously, this task concerns mathematics, which we propose to study.

Secondly, a big problem of the modern school is that students are often offered answers to questions that they did not ask. Therefore, students' motivation to study is not always high. We are convinced that most teachers constantly hear questions from students in classes like: „Why do we need to study this? Where would we need it in real life?" It is clear that the material of not every topic in mathematics is directly used in everyday life, but when creating this textbook, we tried to find ways to demonstrate such applications. Therefore, the textbook is built in the form of a dialogue between students and teachers. At the beginning of each section, seventh-graders Petryk and Tetianka discuss a real-life situation, which ultimately motivates them to study the material of this section. The teacher tries to answer the children's questions, presenting the theoretical material in an accessible, but at the same time mathematically correct.

To consolidate the theoretical material, we offer students exercises and tasks of four levels of difficulty: initial, medium, sufficient and high. At the end

of each item there are control questions from the theory "Check yourself!" and additional information for interested students offered by grandfather Taras, a retired mathematics teacher. Tasks for repetition and preparation for studying new material will also be useful for the teacher's work, here we included tasks from the mathematics course of grades 1-6 and previously studied material of grade 7, as well as propaedeutic tasks for the material of the following points. For each section of the textbook, we provide training test tasks of various forms that allow students to prepare for the thematic control work, and the section ends with tasks of increased complexity, as well as final flowcharts "The main points of the section".

Thirdly, the content of the 7th grade mathematics course is mostly traditional. However, following the ideology of the model program [2], it is supplemented with material that deals with the logical foundations of reasoning, probability and statistics, as well as elements of space geometry. Indeed, the usual 7th grade geometry course is quite difficult for students, since they immediately have to deal with the proofs of abstract statements relating to the same abstract concepts. And what are concepts and their meanings, what is a statement and why and how exactly can it be proved - no one has explained all this to students before in mathematics lessons.

In mathematics textbooks of grades 1-6, strict definitions are mostly not given, and proofs are mostly replaced by heuristic considerations. Considering the age-related psychological and physiological characteristics of students, this is natural, but the formation of abstract thinking sooner or later has to start with something. That is why the traditional geometric material, which concerns the simplest geometric shapes, is preceded by the section "Concepts and their definitions, assertions and their proofs".

In order to eliminate the current gap in the study of spatial geometric figures, we propose to consider spatial figures simultaneously with flat geometric figures, on the study of which the main emphasis is placed. Indeed, in grades 5-6, children are already familiar with spatial geometric bodies: a cube, a rectangular parallelepiped, a prism, a pyramid, a cylinder, a cone, a sphere. If this material is not repeated at all in the mathematics course of grades 7-9, then, obviously, certain problems with the formation of students' spatial imagination may arise in high school. In order to avoid such problems, the 7th grade mathematics course offers a section called "Plane Models of Polyhedra", in which, in particular, it is planned to make paper models of a cube, a rectangular parallelepiped, a triangular and a quadrilateral pyramid.

An important expansion of the content of the 7th grade mathematics course is the strengthening of the probabilistic-statistical line, which is traditional for all modern school mathematics courses in the world. Nowadays, the ability to adequately perceive and process statistical data is a necessary condition for personal adaptation to the real world, which is why the textbook includes the

section “Statistical probabilities”. It lists the main stages of statistical researches, which seventh-graders are offered to implement using the simplest examples from everyday life. Also, this chapter repeats the classical approach to calculating the probabilities of random events, already known to students from grades 5-6, and also considers a more popular in practice statistical approach to calculating probabilities.

Finally, *fourthly*, our mathematics textbook for grade 7 takes into account the modern development of technology and the changing lifestyle and thinking style of modern students. It is obvious that the current generation of seventh-graders is significantly different from their peers even at the beginning of the 21st century, not to mention earlier periods. Therefore, many modern students find traditional textbooks, aimed mainly at readers, which contain large volumes of text from the author, boring and uninteresting.

Most of today’s teenagers, so to speak, live with smartphones in hand, and artificial restrictions on the use of mobile devices, messengers, social networks and other technological innovations, in our opinion, can only lead to rejection and additional tension in communication between students and teachers. We believe that it is worthwhile not to limit or refuse, but to widely use these technologies for teaching mathematics. That is why the theoretical material of the textbook is presented in the form of a chat familiar to students, and hyperlinks and QR codes embedded in its text will simplify and speed up access to the necessary data.

It is also important to understand that with the expansion of the content of the 7th grade mathematics course, which was discussed above, it is necessary to ensure that the students are not overloaded. This will be facilitated by the systematic use of competencies already acquired by students during the study of mathematics in grades 1-6. Also, avoiding the overload of seventh graders is achieved by reducing routine arithmetic operations, graphic constructions and algebraic transformations, which are rarely used in practice and can be performed using modern software tools. For example, our tutorial suggests:

- during studying the transformations of whole expressions, limit yourself to only the simplest transformations that will allow you to solve linear equations and equations that reduce to them;
- during studying the formulas of abbreviated multiplication, limit yourself only to the formulas of the difference of squares, the square of the sum, and the square of the difference;
- use graphing calculators to construct graphs of functions and study their properties.

In our opinion, the pedagogically balanced use of modern technologies can enrich the process of learning mathematics and contribute to the positive emotional perception of the relevant material by students.

We will show the methodological features of studying one of the sections according to this textbook. Let’s take as an example a new section, the study

of which is introduced for the first time in the 7th grade within the framework of the "New Ukrainian School" project. The section is entitled "Concepts and their definitions, assertions and their proof".

The main idea of the introduction of this section is to prepare and adapt students to the next study of the material previously studied in the geometry course. Traditionally, geometric material is taught in school using axiomatic theory. That is, undefined (primitive) concepts of geometry (point, line, plane) and undefined relations (belong, lie between, lie on one side) are considered first. Further, the definitions of other geometric shapes (segment, ray, angle, triangle, etc.) are given using undefined concepts and relations. Statements are formulated about these abstract objects, some of which are accepted without proof (axioms), and others are proved on the basis of logical considerations (theorems).

The already described scheme for studying geometric material suggests that it will not be easy for seventh graders to master it, and the pedagogical experience of the authors of the textbook only further confirms this. The thing is that students immediately have to deal with abstract concepts and formal logical reasoning about them. And this (perhaps, with certain exceptions) no one ever taught them. In addition, due to age characteristics, the abstract thinking of seventh-graders is just beginning to form. The material of this chapter should contribute to this formation.

At the same time, we note that we do not set ourselves the goal of presenting the basics of logic absolutely strictly and completely correctly from a scientific point of view. Apparently, for most students of the 7th grade, this level of presentation is inaccessible for perception. In addition, this section is not designed to complicate and formalize the further study of geometric material, but on the contrary - to simplify it through the use of analogies. Students should understand that they have already given the definitions of the concepts, and the truth of the statements has also been repeatedly established both in everyday life and when studying the mathematics course of grades 1-6. In chapter 4, we aim to show students that logical reasoning (including formulating the meanings of concepts and proving the truth of statements) is inherent in all spheres of human life and does not apply only to certain abstract geometric objects - points, lines, segments, triangles, circles, etc.

The first paragraph of the chapter deals with concepts and their definitions. From the specific examples of Petryk and Tatyanka, students begin to understand that not all concepts used both in everyday life and in science can be given a definition, because the definitions of new concepts are mostly given through other previously defined concepts, so such a chain cannot last indefinitely. Therefore, it is natural not to give definitions to individual concepts, but only to model and describe them in an understandable way. It is worth stimulating the students in the lesson to build their own chains of concepts until they reach the need to agree to make some concept undefined.

After the students realize the need for undefined concepts and, based on concrete examples from everyday life and previously studied concepts of the mathematics course, understand how they can be defined, it is worth giving examples of a point, a straight line, and a plane as the main undefined concepts of geometry. This will be a kind of propaedeutic stuff for studying the material of the next chapter. It is important that students understand the difference between the concept itself and the term that denotes it. For this, you can make the following analogy: the concept is what lies in the box, and the term is what is written on this box. Examples of different terms referring to the same concept will also help students better understand the difference.

It is convenient to depict the relationship between the volumes of concepts using Venn diagrams. Students already performed such tasks in the 5th grade, without focusing on the relevant theoretical component. Therefore, this material is already known to them, and therefore should not cause significant difficulties. For gifted students, it is important to emphasize the relationship between the scope and the content of the concept: the greater the content of the concept (the number of properties that describe it), the smaller its scope, and vice versa - the smaller the content of the concept, the greater its scope. At the same time, in our opinion, students should not be required to reproduce the meanings of the content and scope of the concept, it is enough for students to understand what it is with concrete examples.

We additionally emphasize that the material of this paragraph should not be overly formalized. Seventh-graders should master this material, in fact, in a playful way, having a large space for creative search, expressing their own (perhaps not always successful) suggestions for marking concepts from everyday life. The teacher should act as a moderator, helping students to avoid logical errors and inaccuracies in writing.

The second paragraph of the chapter examines the statements and their proofs. Please note that the definition of the concept of „assertion" is given after considering specific examples of statements from everyday life. We do not recommend requiring all students to memorize this definition, instead, it is worth asking students to give examples of their own statements related to everyday life and try to justify their truth or falsity. At the same time, it is desirable to pay attention to the arguments that students give in favor of their position. In this case, it is again useful to give the seventh graders some freedom and the right to make mistakes in their argumentation, acting as a moderator. However, at the end of the lesson, the teacher should express his own reasoned opinion about the reasoning given by the students, which indicates the logical errors in these reasoning (if there were any).

We believe that emphasis should be placed on the fact that the truth of many statements is determined *by agreement*. At the same time, this applies to the sphere of communication, ethical norms, and legal norms. However, na-

turally, it is impossible to agree on the truth or falsity of absolutely all or even most of the statements, because this at least creates technical inconveniences in use due to the cumbersomeness of the search for the appropriate agreement. Therefore, it is natural that the truth is agreed upon only for a certain minimum number of statements (they are called axioms), and the truth or falsity of other statements is substantiated with the help of logical reasoning (they are called theorems). This makes it possible to reduce the volume of legislative documents, rules of internal procedure, etc. Such documents become more convenient to use, but there is a need to develop logical reasoning that will allow a limited number of agreements to make a decision in a case that does not directly coincide with any of them.

The laws of logic are used to carry out logical reasoning. We do not recommend requiring students to formally memorize these laws, instead, it is important to provide enough concrete examples in the lessons to achieve an understanding of these laws. In particular, it is important that students understand the difference between statements that are the opposite and controversy of a given statement. A sufficient number of statements from everyday life should be given to illustrate that a given statement can be given many opposite statements, but only one of them will be the controversy. This is useful for further understanding of the essence of the method of proof from its controversy, which is based on the law of exclusion of the third.

After considering statements from everyday life, it is natural to give examples of statements already known to students from the 1-6 grade mathematics course, showing which of them are axioms and which are theorems. For example, the laws of addition and multiplication (commutative, conjunctive, and distributive) may well be considered axioms, but all statements about reduced multiplication formulas are obviously theorems, the proof of which consists in the consistent application of the mentioned laws. By this, we bring the seventh graders to the conclusion that all the material offered in chapter 4 is actually, in fact, not fundamentally new, it is only necessary to place certain accents in the already known.

Conclusions. Updating the content of the academic discipline “Mathematics” in secondary schools is a complex and long-term process. We are convinced that revolutionary changes in this area will not lead to immediate improvement and ensuring the proper quality of education. In our opinion, it is necessary to follow a long evolutionary path, gradually updating the entire methodical system of teaching mathematics at school: goals, content, methods, means and organizational forms of education. In our model program and the 7th grade textbook discussed in the article, such an evolutionary approach is implemented. We keep the content about 80% traditional, but make sure to add an innovative 20% that hasn’t been studied before. We are changing the traditional approaches to the textbook (apparently these changes are difficult

to measure in percentages), but we are also doing it gradually, taking into account the characteristics of the new generation of teachers and students. We are convinced that we are on the right path, but we are always open to constructive criticism and discussions in the chosen direction.

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