ON SOLUTIONS OF SOME DIOPHANTINE EQUATIONS

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Abstract

În această lucrare se arată că ecuațiile Diofantice: $2^x + 67^y = z^2$, $2^x + 167^y = z^2$ și $2^x + 1367^y = z^2$ au exact una și două (x, y, z) în mulțimea numerelor întregi nenegative {(3,0,3)},{(3,0,3),(1,1,13)} și {(3,0,3), (1,1,37)}, respectiv.

Cuvinte cheie: Ecuații Diofantice exponențiale, soluții întregi.

1. Introduction

One important topic in number theory is the study of Diophantine equations, equations in which only integer solutions are permitted. The field of Diophantine equations is ancient, vast, and no general method exist to decide whether a given Diophantine equation has any solutions, or how many solutions. The famous general equation $p^x + q^y = z^2$ has many forms. The literature contains a very large number of articles on non–linear such individual equations involving particular primes and powers of all kinds.

In this paper, we study equations: $2^x + 67^y = z^2$,$2 + 167^y = z^2$ and $2^x + 1367^y = z^2$ where x, y, z are non–negative integer. One could cite here many articles on the equation $p^x + q^y = z^2$. We provide here only a small number of related equations which include the prime 2 in particular, such as [1,2,3,4,5,6].

2. Solutions of the equations: $2^x + 67^y = z^2, 2^x + 167^y = z^2$ and $2^x + 1367^y = z^2$

Theorem 1. The Diophantine equation $2^x + 67^y = z^2$ has exactly one non–negative integer solutions $(3,0,3)$.

Proof. We consider several cases.

Case 1. For $x = 0$, then we have the Diophantine equation $1 + 67^y = z^2$or$(z - 1)(z + +1) = 67^y$ where $z - 1 = 67^t$ and $z + 1 = 67^{y-t}$, $y > 2t, t \in \mathbb{N}$. From here, we obtain $67^t(67^{y-t} - 1) = 2$ where $t = 0$ and $67^y = 3$, which is impossible.

Case 2. For $y = 0$, then we have the Diophantine equation $2^x + 1 = z^2$ or $(z - 1)(z + +1) = 2^x$ where $z - 1 = 2^s$and $z + 1 = 2^{x-s}$, $x > 2s, s \in \mathbb{N}$. From here, we obtain$2^s(2^{x-2s} - -1) = 2$ where $s = 1$ and $2^{x-2} = 2$, that is $s = 1$ and $x = 3$.

Case 3. For $x = 2n, n \in \mathbb{N}^*$, then we have the Diophantine equation $2^{2n} + 67^y = z^2$or $(z - 2^n)(z + 2^n) = 67^y$ where $z - 2^n = 67^q$and $z + 2^n = 67^{y-q}$, $y > 2q, q \in \mathbb{N}$. From here, we obtain $67^q(67^{y-2q} - 1) = 2^{n+1}$which implies $q = 0$ and $67^y - 1 = 2^{n+1}$. If $y \geq 1$ we have $67\delta = 2^{n+1}$, $\delta \in \mathbb{N}^*$, here it results that 13 divide 2^{n+1},which is impossible.

Case 4. For $y = 2m, m \in \mathbb{N}^*$, then we have the Diophantine equation $2^x + 67^{2m} = z^2$ or $(z - 67^m)(z + 67^m) = 2^x$ where $z - 67^m = 2^\beta$ and $z + 67^m = 2^{x-\beta}$, $x > 2\beta, \beta \in \mathbb{N}$. From here, we obtain $2^\beta(2^{x-2\beta} - 1) = 2 \cdot 67^m$ which implies $\beta = 1$ and $2^{x-2} - 1 = 53^m$. Using modulo 67 we have $x - 2 = 66r, r \in \mathbb{N}^*$ or $2^{66r} - 1 = 67^m$ or $4^{33r} - 1 = 67^m$ from where we obtain $3l = 67^m, l \in \mathbb{N}^*$, so 3 divides 67 which is impossible.

Case 5. Let x, y is odd. Because z is odd, then we have $z = 2p + 1$, it results $z^2 = 4p(p + 1) + 1 \equiv 1(mod8)$. If $x \geq 3$ and odd we have $2^x \equiv 0(mod8)$, and if y odd we have $67^{2k+1} = 67^{2k} \cdot 67 \equiv 3(mod8)$. From here, we obtain $2^x + 67^y \equiv 3(mod8)$, which is impossible, because $z^2 \equiv 1(mod8)$.

If $x = 1, y = 1$, then $2^1 + 67^1 = 69$. In concluding the Diophantine equation has a solution $(3,0,3)$. This proves the theorem.

Theorem 2. The Diophantine equation $2^x + 167^y = z^2$ has exactly two non–negative integer solutions $(3,0,3)$ and $(1,1,13)$.

Proof. We consider several cases.

Case 1. For $x = 0$, then we have the Diophantine equation $1 + 167^y = z^2$ or $(z - 1)(z + +1) = 167^y$ where $z - 1 = 167^k$ and $z + 1 = 167^{y-t}, y > 2k, k \in \mathbb{N}$. From here, we obtain $167^t(167^{y-t} - 1) = 2$ where $t = 0$ and $167^y = 3$, which is impossible.

Case 2. For $y = 0$, then we have the Diophantine equation $2^x + 1 = z^2$ or $(z - 1)(z + +1) = 2^x$ where $z - 1 = 2^q$ and $z + 1 = 2^{x-q}$, $x > 2q, q \in \mathbb{N}$. From here, we obtain $2^q(2^{x-2q} - 1) = 2$ where $s = 1$ and $2^{x-2} = 2$, that is $q = 1$ and $x = 3$.

Case 3. For $x = 2n, n \in N^*$, then we have the Diophantine equation $2^{2n} + 167^y = z^2$ or $(z - 2^n) \cdot (z + 2^n) = 167^y$ where $z - 2^n = 167^s$ and $z + 2^n = 167^{y-s}$, $y > 2s, s \in N$. From here, we obtain $167^s(727^{y-2s} - 1) = 2^{n+1}$ which implies $s = 0$ and $167^y - 1 = 2^{n+1}$. If $y \geq 1$ we have $166\gamma = 2^{n+1}, \gamma \in \mathbb{N}^*$, here it results that 83 divide 2^{n+1},which is impossible.

Case 4. For $y = 2m, m \in \mathbb{N}^*$, then we have the Diophantine equation $2^x + 167^{2m} = z^2$ or $(z - 167^m) \cdot (z + 167^m) = 2^x$ where $z - 167^m = 2^\beta$ and $z + 167^m = 2^{x-\beta}, x > 2\beta, \beta \in \mathbb{N}$. From here, we obtain $2^\beta(2^{x-2\beta} - 1) = 2 \cdot 167^m$ which implies $\beta = 1$ and $2^{x-2} - 1 = 167^m$. Using modulo 167 we have $x - 2 = 166r, r \in \mathbb{N}^*$ or $2^{166r} - 1 = 167^m$ or $4^{83r} - 1 = = 167^m$ from where we obtain $3l = 167^m, l \in \mathbb{N}^*$, so 3 divides 167 which is impossible.

Case 5. Let x, y is odd. Becauze z is odd, then we have $z = 2p + 1$, it results $z^2 = 4p(p + 1) + 1 \equiv 1(mod 8)$. If $x \geq 3$ and odd we have $2^x \equiv 0(mod 8)$, and if y odd we have $167^{2k+1} = 167^{2k} \cdot 167 \equiv 7(mod 8)$. From here, we obtain $2^x + 167^y \equiv 7(mod 8)$, which is imposible, because $z^2 \equiv 1(mod 8)$.

If $x = 1, y = 1$, then $2^1 + 167^1 = 169 = 13^2$. In concluding the Diophantine equation has a solution$(3,0,3)$ and $(1,1,13)$.This proves the theorem.

Theorem 3. The Diophantine equation $2^x + 1367^y = z^2$ has exactly two non–negative integer solutions $(3,0,3)$ and $(1,1,37)$.

Proof. We consider several cases.

Case 1. For $x = 0$, then we have the Diophantine equation $1 + 1367^y = z^2$ or $(z - -1)(z + 1) = 1367^y$ where $z - 1 = 1367^v$ and $z + 1 = 1367^{y-v}$, $y > 2t = v, v \in \mathbb{N}$. From here, we obtain $1367(1367^{y-v} - 1) = 2$ where $v = 0$ and $1367^y = 3$, which is impossible.

Case 2. For $y = 0$, then we have the Diophantine equation$2^x + 1 = z^2$or $(z - 1)(z + 1) = = 2^x$ where $z - 1 = 2^s$and$z + 1 = 2^{x-s}, x > 2s, s \in \mathbb{N}$. From here, we obtain$2^s(2^{x-2s} - 1) = = 2$ where $s = 1$ and $2^{x-2} = 2$, that is $s = 1$ and $x = 3$.

Case 3. For $x = 2n, n \in N^*$, then we have the Diophantine equation $2^{2n} + 1367^y = z^2$or $(z - 2^n) \cdot (z + 2^n) = 1367^y$ where $z - 2^n = 1367^q$ and $z + 2^n = 1367^{y-q}$, $y > 2q, q \in \mathbb{N}$. From here, we obtain $1367^q(1367^{y-2q} - 1) = 2^{n+1}$which implies $q = 0$ and $23^y - 1 = 2^{n+1}$. If $y \geq 1$ we have $1366t = 2^{n+1}t, \in \mathbb{N}^*$, here it results that 1366 divide 2^{n+1},which is impossible.

Case 4. For $y = 2m, m \in \mathbb{N}^*$, then we have the Diophantine equation $2^x + 1367^{2m} = z^2$ or $(z - 1367^m) \cdot (z + 1367^m) = 2^x$ where $z - 1367^m = 2^\beta$ and$z + 1367^m = 2^{x-\beta}, x > 2\beta$, $\beta \in N$. From here, we obtain $2^\beta(2^{x-2\beta} - 1) = 2 \cdot 1367^m$ which implies $\beta = 1$ and $2^{x-2} - 1 =$

$= 1367^m$. Using modulo 1367 we have $x - 2 = 1366r, r \in \mathbb{N}^*$ or $2^{1366s} - 1 = 23^m$ or $4^{683s} - 1 = 1367^m$ from where we obtain $3d = 1367^m, d \in \mathbb{N}^*$, so 3 divides 1367 which is impossible.

Case 5. Let x, y is odd. Becauze z is odd, then we have $z = 2p + 1$, it results $z^2 = 4p(p + 1) + 1 \equiv 1(mod8)$. If $x \geq 3$ and odd we have $2^x \equiv 0(mod8)$, and if y odd we have $1367^{2k+1} = 1367^{2k} \cdot 1367 \equiv 7(mod8)$. From here, we obtain $2^x + 1367^y \equiv 7(mod8)$, which is imposible, because $z^2 \equiv 1(mod8)$.

If $x = 1, y = 1$, then $2^1 + 1367^1 = 1369 = 37^2$. In concluding the Diophantine equation has a solution $(3,0,3)$ and (1,1,37). This proves the theorem.

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ELABORAREA ȘI INTERGRAREA RESURSELOR AUDIO-VIDEO ÎN CADRUL MANUALELOR DIGITALE

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Abstract

The use of Audio Visual device in teaching and learning has increased in the past few years due to the technological developments. Audio-visual resources can greatly enrich the books bringing to life teaching and learning opportunities and giving the potential to bring the outside world into the classroom broadening and enhancing the learners’ experience. The paper presents some popular softwares for creating audio-video resources.

Key-words: multimedia educational resources, digital manuals, audio, video.

„Manualele sunt cartea de vizită a unui sistem de educaţie”.
Dr, Olimpius Istrate, Universitatea din București (2013)

Tendințele educației contemporane nu se mai focusează pe memorarea informaţiei, ci pe abordarea ei într-o manieră creativă, originală. Interfeţele grafice alcătuite din imagini, diagrame, hărţi şi materiale video ajung să înlocuiască treptat materialele de studiu imprimate, pentru că nenumărate studii în domeniu au confirmat impactul deosebit de puternic al stimulilor vizuali asupra procesului de învăţare. Astfel, potrivit expertului în psihologia educaţională Glasser [1], „Omul învaţă: 10% din ceea ce citeşte; 20% din ceea ce aude; 30% din ceea ce vede; 50% din ceea ce aude şi vede; 70% din ceea ce discută cu alţii; 80% din propria experienţă; 95% din ceea ce învaţă pe alţii”.